Exercise 1.2.17 *Determine if the system is consistent. If so, is the solution unique?*

$$\begin{aligned} x+2y+z-w&=2\\ x-y+z+w&=1\\ 2x+y-z&=1\\ 4x+2y+z&=5 \end{aligned}$$

Exercise 1.2.18 *Determine if the system is consistent. If so, is the solution unique?*

$$\begin{aligned} x+2y+z-w&=2\\ x-y+z+w&=0\\ 2x+y-z&=1\\ 4x+2y+z&=3 \end{aligned}$$

Exercise 1.2.20 *Row reduce the following matrix to obtain the row-echelon form. Then continue to obtain the reduced row-echelon form.*

$$\left[\begin{array}{rrrr} 2 & -1 & 3 & -1\\ 1 & 0 & 2 & 1\\ 1 & -1 & 1 & -2 \end{array}\right]$$

Exercise 1.2.21 *Row reduce the following matrix to obtain the row-echelon form. Then continue to obtain the reduced row-echelon form.*

$$\left[\begin{array}{rrrr} 0 & 0 & -1 & -1\\ 1 & 1 & 1 & 0\\ 1 & 1 & 0 & -1 \end{array}\right]$$

Exercise 1.2.22 *Row reduce the following matrix to obtain the row-echelon form. Then continue to obtain the reduced row-echelon form.*

$$\left[\begin{array}{cccc} 3 & -6 & -7 & -8 \\ 1 & -2 & -2 & -2 \\ 1 & -2 & -3 & -4 \end{array}\right]$$

Exercise 1.2.23 *Row reduce the following matrix to obtain the row-echelon form. Then continue to obtain the reduced row-echelon form.*

$$\left[\begin{array}{cccc} 2 & 4 & 5 & 15 \\ 1 & 2 & 3 & 9 \\ 1 & 2 & 2 & 6 \end{array}\right]$$

Exercise 1.2.24 *Row reduce the following matrix to obtain the row-echelon form. Then continue to obtain the reduced row-echelon form.*

$$\left[\begin{array}{cccc} 4 & -1 & 7 & 10 \\ 1 & 0 & 3 & 3 \\ 1 & -1 & -2 & 1 \end{array}\right]$$

Exercise 1.2.25 *Row reduce the following matrix to obtain the row-echelon form. Then continue to obtain the reduced row-echelon form.*

$$\left[\begin{array}{cccc} 3 & 5 & -4 & 2 \\ 1 & 2 & -1 & 1 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

Exercise 1.2.26 *Row reduce the following matrix to obtain the row-echelon form. Then continue to obtain the reduced row-echelon form.*

$$\left[\begin{array}{cccc} -2 & 3 & -8 & 7 \\ 1 & -2 & 5 & -5 \\ 1 & -3 & 7 & -8 \end{array}\right]$$

Exercise 1.2.27 *Find the solution of the system whose augmented matrix is*

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 1 & 3 & 4 & 2 \\ 1 & 0 & 2 & 1 \end{array}\right]$$

Exercise 1.2.28 *Find the solution of the system whose augmented matrix is*

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ 3 & 2 & 1 & 3 \end{array}\right]$$

Exercise 1.2.30 *Find the solution of the system whose augmented matrix is*

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 1 & 2 & 0 & 0 & 1 & 3 \\ 1 & 0 & 1 & 0 & 2 & 2 \end{array}\right]$$

Exercise 1.2.31 *Find the solution of the system whose augmented matrix is*

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 2 & 0 & 0 & 1 & 3 \\ 1 & -1 & 2 & 2 & 2 & 0 \end{array}\right]$$

Exercise 1.2.32 *Find the solution to the system of equations,* $7x+14y+15z=22$, $2x+4y+3z=5$, *and* $3x+6y+10z=13$.

Exercise 1.2.33 *Find the solution to the system of equations,* $3x-y+4z=6$, $y+8z=0$, *and* $-2x+y=-4$.

Exercise 1.2.34 *Find the solution to the system of equations,* $9x-2y+4z=-17$, $13x-3y+6z=-25$, *and* $-2x-z=3$.

Exercise 1.2.35 *Find the solution to the system of equations,* $65x+84y+16z=546$, $81x+105y+20z=682$, *and* $84x+110y+21z=713$.

Exercise 1.2.36 *Find the solution to the system of equations,* $8x+2y+3z=-3, 8x+3y+3z=-1$, *and* $4x+y+3z=-9$.

Exercise 1.2.37 *Find the solution to the system of equations,* $-8x+2y+5z=18, -8x+3y+5z=13$, *and* $-4x+y+5z=19$.

Exercise 1.2.38 *Find the solution to the system of equations,* $3x-y-2z=3$, $y-4z=0$, *and* $-2x+y=-2$.

Exercise 1.2.39 *Find the solution to the system of equations,* $-9x+15y=66, -11x+18y=79$, $-x+y=4$, *and* $z=3$.

Exercise 1.2.46 *Find the rank of the following matrix.*

$$\begin{bmatrix} 4 & -16 & -1 & -5 \\ 1 & -4 & 0 & -1 \\ 1 & -4 & -1 & -2 \end{bmatrix}$$

Exercise 1.2.47 *Find the rank of the following matrix.*

$$\begin{bmatrix} 3 & 6 & 5 & 12 \\ 1 & 2 & 2 & 5 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

Exercise 1.2.48 *Find the rank of the following matrix.*

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 3 \\ 1 & 4 & 1 & 0 & -8 \\ 1 & 4 & 0 & 1 & 2 \\ -1 & -4 & 0 & -1 & -2 \end{bmatrix}$$

Exercise 1.2.49 *Find the rank of the following matrix.*

$$\begin{bmatrix} 4 & -4 & 3 & -9 \\ 1 & -1 & 1 & -2 \\ 1 & -1 & 0 & -3 \end{bmatrix}$$

Exercise 1.2.50 *Find the rank of the following matrix.*

$$\begin{bmatrix} 2 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 7 \\ 1 & 0 & 0 & 1 & 7 \end{bmatrix}$$

Exercise 1.2.51 *Find the rank of the following matrix.*

$$\begin{bmatrix} 4 & 15 & 29 \\ 1 & 4 & 8 \\ 1 & 3 & 5 \\ 3 & 9 & 15 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

no sol'n

Ex. 1.2.17

$$\begin{cases} x + 2y + z - w = 2 \\ x - y + z + w = 1 \\ 2x + y - z = 1 \\ 4x + 2y + z = 5 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 1 & -1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 0 & 1 \\ 4 & 2 & 1 & 0 & 5 \end{array}\right) \quad \begin{array}{l} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - 4R_1 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -3 & 0 & 2 & -1 \\ 0 & -3 & -3 & 2 & -3 \\ 0 & -6 & -3 & 4 & -3 \end{array}\right) \quad \begin{array}{l} \\ \\ R_3 \to R_3 - R_2 \\ R_4 \to R_4 - 2R_2 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -3 & 0 & 2 & -1 \\ 0 & 0 & -3 & 0 & -2 \\ 0 & 0 & -3 & 0 & -1 \end{array}\right) \quad \begin{array}{l} \\ \\ \\ R_4 \to R_4 - R_3 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -3 & 0 & 2 & -1 \\ 0 & 0 & -3 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

(S) has no sol'n

6. 1.2.18

$$\begin{cases} x + 2y + z - w = 2 \\ x - y + z + w = 0 \\ 2x + y - z = 1 \\ 4x + 2y + z = 3 \end{cases}$$

(1)

$$= \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 & 1 \\ 4 & 2 & 1 & 0 & 3 \end{array}\right) \begin{array}{l} \\ R_2 \to R_2 - R_1 \quad (2) \\ R_3 \to R_3 - 2R_1 \quad (3) \\ R_4 \to R_4 - 4R_1 \end{array}$$

(4)

$$= \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -3 & 0 & 2 & -2 \\ 0 & -3 & -3 & 2 & -3 \\ 0 & -6 & -3 & 4 & -5 \end{array}\right) \begin{array}{l} \\ \\ R_3 \to R_3 - R_2 \quad (5) \\ R_4 \to R_4 - 2R_2 \end{array}$$

(6)

$$= \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -3 & 0 & 2 & -2 \\ 0 & 0 & -3 & 0 & -1 \\ 0 & 0 & -3 & 0 & 8 \end{array}\right) \begin{array}{l} \\ \\ \\ R_4 \to R_4 - R_3 \end{array}$$

$$= \left(\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 2 \\ 0 & -3 & 0 & 2 & -2 \\ 0 & 0 & -3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 9 \end{array}\right)$$

(S) has no sol^n

1.2.20 $\begin{bmatrix} 2 & -1 & 3 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 1 & -2 \end{bmatrix}$ $R_1 \leftrightarrow R_2$

$= \begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & -1 & 3 & -1 \\ 1 & -1 & 1 & -2 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$= \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & -1 & -1 & -3 \end{bmatrix}$ $R_3 \to R_3 - R_2$

$= \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \to -R_2$

$= \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ in r.e.f. & in r.r.e.f

1.2.21 $\begin{bmatrix} 0 & 0 & -1 & -1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$ $R_1 \leftrightarrow R_2$

$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & -1 \end{bmatrix}$ $R_1 \leftrightarrow R_3$

$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix}$ $R_2 \to R_2 - R_1$

$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix}$ $R_3 \to R_3 - R_2$

$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ r.e.f

$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \to -R_2$

$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_1 \to R_1 - R_2$

$= \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ r.r.e.f

10,000
10,000
10,000

1.2.22 $\begin{bmatrix} 3 & -6 & -7 & -8 \\ 1 & -2 & -2 & -2 \\ 1 & -2 & -3 & -4 \end{bmatrix}$ $R_1 \to \frac{1}{3} R_1$

$\to \begin{bmatrix} 1 & -2 & -7/3 & -8/3 \\ 1 & -2 & -2 & -2 \\ 1 & -2 & -3 & -4 \end{bmatrix}$ $R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$

$\to \begin{bmatrix} 1 & -2 & -7/3 & -8/3 \\ 0 & 0 & 1/3 & 2/3 \\ 0 & 0 & -2/3 & -4/3 \end{bmatrix}$ $R_3 \to R_3 + 2R_2$

$\to \begin{bmatrix} 1 & -2 & -7/3 & -8/3 \\ 0 & 0 & 1/3 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \to 3R_2$

$\to \begin{bmatrix} 1 & -2 & -7/3 & -8/3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ r.e.f.

$\to \begin{bmatrix} 1 & -2 & -7/3 & -8/3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_1 \to R_1 + \frac{7}{3} R_2$

$-8/3 + 7/3(2)$

$-8+14$

$6/3 = 2$

$\to \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ r.r.e.f.

1.2.23

$$\begin{bmatrix} 2 & 4 & 5 & 15 \\ 1 & 2 & 3 & 9 \\ 1 & 2 & 2 & 6 \end{bmatrix} \qquad R_1 \to \tfrac{1}{2} R_1$$

$$\begin{bmatrix} 1 & 2 & 5/2 & 15/2 \\ 1 & 2 & 3 & 9 \\ 1 & 2 & 2 & 6 \end{bmatrix} \qquad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$3 - 5/2$

$6 - 5/2$

$$\begin{bmatrix} 1 & 2 & 5/2 & 15/2 \\ 0 & 0 & 1/2 & 3/2 \\ 0 & 0 & -1/2 & -3/2 \end{bmatrix} \qquad R_3 \to R_3 + R_2$$

$9 - 15/2$

$3/2$

$$\begin{bmatrix} 1 & 2 & 5/2 & 15/2 \\ 0 & 0 & 1/2 & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_2 \to 2R_2$$

$2 - 5/2$

$-1/2$

$6 - 15/2$

$12 - 15/2$

$$\begin{bmatrix} 1 & 2 & 5/2 & 15/2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{r.e.f.}$$

$3/2$

$$\begin{bmatrix} 1 & 2 & 5/2 & 15/2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_1 \to R_1 - \tfrac{5}{2} R_2$$

$\frac{5}{2}(3)$

$15/2$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{r.r.e.f.}$$

1.2.24

$$\begin{bmatrix} 4 & -1 & 7 & 10 \\ 1 & 0 & 3 & 3 \\ 1 & -1 & -2 & 1 \end{bmatrix} \quad R_1 \to \frac{1}{4} R_1$$

$$\to \begin{bmatrix} 1 & -\frac{1}{4} & \frac{7}{4} & \frac{5}{2} \\ 1 & 0 & 3 & 3 \\ 1 & -1 & -2 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$3 - \frac{7}{4}$

$12 - 7$

$3 - \frac{5}{2}$

$\frac{1}{2}$

$$\to \begin{bmatrix} 1 & -\frac{1}{4} & \frac{7}{4} & \frac{5}{2} \\ 0 & \frac{1}{4} & \frac{5}{4} & \frac{1}{2} \\ 0 & -\frac{3}{4} & -\frac{15}{4} & -\frac{3}{2} \end{bmatrix} \quad R_3 \to R_3 + 3R_2$$

$-1 + \frac{1}{4}$

$-2 - \frac{7}{4}$

$\frac{-8 - 7}{4}$

$1 - \frac{5}{2}$

$$\to \begin{bmatrix} 1 & -\frac{1}{4} & \frac{7}{4} & \frac{5}{2} \\ 0 & \frac{1}{4} & \frac{5}{4} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \to 4R_2$$

$$\to \begin{bmatrix} 1 & -\frac{1}{4} & \frac{7}{4} & \frac{5}{2} \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{r.e.f.}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{4} & \frac{7}{4} & \frac{5}{2} \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 + \frac{1}{4} R_2$$

$\frac{7}{4} + \frac{1}{4}(5)$

$\frac{12}{4} = 3$

$$\to \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{r.r.e.f.}$$

$\frac{5}{2} + \frac{1}{2}(2)$

$\frac{6}{2} = 3$

1.2.25 $\begin{bmatrix} 3 & 5 & -4 & 2 \\ 1 & 2 & -1 & 1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $R_1 \to \frac{1}{3} R_1$

$\to \begin{bmatrix} 1 & \frac{5}{3} & -\frac{4}{3} & \frac{2}{3} \\ 1 & 2 & -1 & 1 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\to \begin{bmatrix} 1 & \frac{5}{3} & -\frac{4}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} \end{bmatrix}$ $R_3 \to R_3 + 2R_2$

$\to \begin{bmatrix} 1 & \frac{5}{3} & -\frac{4}{3} & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \to 3R_2$

$\to \begin{bmatrix} 1 & \frac{5}{3} & -\frac{4}{3} & \frac{2}{3} \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ r.e.f.

$\to \begin{bmatrix} 1 & \frac{5}{3} & -\frac{4}{3} & \frac{2}{3} \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_1 \to R_1 - \frac{5}{3} R_2$

$\to \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ r.r.e.f.

1.2.26

$$\begin{bmatrix} -2 & 3 & -8 & 7 \\ 1 & -2 & 5 & -5 \\ 1 & -3 & 7 & -8 \end{bmatrix} \quad R_1 \to -\tfrac{1}{2} R_1$$

$$\to \begin{bmatrix} 1 & -\tfrac{3}{2} & 4 & -\tfrac{7}{2} \\ 1 & -2 & 5 & -5 \\ 1 & -3 & 7 & -8 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$-2+\frac{3}{2}$

$\frac{-4+3}{2}$

$$\to \begin{bmatrix} 1 & -\tfrac{3}{2} & 4 & -\tfrac{7}{2} \\ 0 & -\tfrac{1}{2} & 1 & -\tfrac{3}{2} \\ 0 & -\tfrac{3}{2} & 3 & -\tfrac{9}{2} \end{bmatrix} \quad R_3 \to R_3 - 3R_2$$

$-5+\frac{7}{2}$

$-10+7$

$-3+\frac{3}{2}$

$\frac{-6+3}{2} = -\frac{3}{2}$

$$\to \begin{bmatrix} 1 & -\tfrac{3}{2} & 4 & -\tfrac{7}{2} \\ 0 & -\tfrac{1}{2} & 1 & -\tfrac{3}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \to -2R_2$$

$-8+\frac{7}{2}$

$-16+7$

$\frac{3}{2}$

$$\to \begin{bmatrix} 1 & -\tfrac{3}{2} & 4 & -\tfrac{7}{2} \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{r.e.f.}$$

$$\begin{bmatrix} 1 & -\tfrac{3}{2} & 4 & -\tfrac{7}{2} \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 + \tfrac{3}{2} R_2$$

$4+\frac{3}{2}(-2)$

$-\frac{7}{2}+\frac{3}{2}(3)$

$-7+9$

$=\frac{2}{2}$

$$\to \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{r.r.e.f.}$$

1.2.27 $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 1 & 3 & 4 & 2 \\ 1 & 0 & 2 & 1 \end{array}\right]$

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ 1 & 0 & 2 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$= \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & -2 & 2 \end{pmatrix} \quad R_3 \to R_3 + 2R_2$$

$$= \underbrace{\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 10 \end{pmatrix}}_{U} \quad \text{upper triangular matrix}$$

$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$E_1 = I_3(R_2 \to R_2 - R_1) = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to E_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_2 = I_3(R_3 \to R_3 - R_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \to E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$E_3 = I_3(R_3 \to R_3 + 2R_2) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \to E_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

$$L = E_1^{-1} \times E_2^{-1} \times E_3^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$A = LU$

$\forall AX = b \iff LUX = b$

$\iff LY = b \;/\; Y = UX$

$$\begin{cases} ① & LY = b \\ ② & UX = Y \end{cases}$$

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ 1 & 0 & 2 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

① $LY = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\iff \begin{bmatrix} y_1 \\ y_1 + y_2 \\ y_1 - 2y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \begin{cases} y_1 = 2 & ① \\ y_1 + y_2 = 2 & ② \\ y_1 - 2y_2 + y_3 = 1 & ③ \end{cases}$$

$y_1 = 2$

sub y_1 in ② $\Rightarrow 2 + y_2 = 2 \Rightarrow y_2 = 0$

sub y_1 & y_2 in ③ $\Rightarrow 2 - 2(0) + y_3 = 1 \Rightarrow y_3 = -1$

$$\Rightarrow Y = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

② $UX = Y$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} x_1 + 2x_2 \\ x_2 + 4x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \Rightarrow \begin{cases} x_1 + 2x_2 = 2 & ① \\ x_2 + 4x_3 = 0 & ② \\ x_3 = -1 & ③ \end{cases}$$

sub x_3 in ② $\Rightarrow x_2 + 4(-1) = 0 \Rightarrow x_2 = 4$

sub x_2 in ① $\Rightarrow x_1 + 2(4) = 2 \Rightarrow x_1 = -6$

$$\Rightarrow X = \begin{bmatrix} -6 \\ 4 \\ -1 \end{bmatrix}$$

1.2.28 $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 2 & 0 & 1 & 1 \\ 3 & 2 & 1 & 3 \end{array}\right]$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$\to \left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & -4 & 1 & -3 \\ 0 & -4 & 1 & -3 \end{array}\right]$ $R_3 \to R_3 - R_2$

$\to \left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & -4 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$ Then (S) has infinitely many sol'ns

Let $z = t$

then $-4y + t = -3 \Rightarrow y = \frac{t+3}{4}$

& $x + 2\left(\frac{t+3}{4}\right) = 2$

$x + \frac{t}{2} + \frac{3}{2} = 2 \Rightarrow x = \frac{1-t}{2}$

$\therefore$ sol'n of (S) is $\left\{ \frac{1-t}{2}, \frac{t+3}{4}, t \right\}$

1.2.30

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 1 & 2 & 0 & 0 & 1 & 3 \\ 1 & 0 & 1 & 0 & 2 & 2 \end{array}\right] \quad \begin{array}{l} R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\to \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 2 & -2 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{array}\right] \quad R_2 \to R_2 - 2R_2$$

$-1-2$

$0-2(2)$

$1-2(1)$

$$\to \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & -2 & -3 & -4 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{array}\right] \quad \begin{array}{l} \to R_3 \to -\frac{1}{2} R_2 \\ \to R_4 \to -R_4 \end{array}$$

$$\to \left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{3}{2} & 2 & \frac{1}{2} \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right] \quad R_4 \to R_4 - R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{3}{2} & 2 & \frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & -3 & -\frac{1}{2} \end{array}\right] \quad R_4 \to -2R_4$$

$1-\frac{3}{2}$

$-\frac{1}{2}$

$-1-2$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{3}{2} & 2 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 6 & 1 \end{array}\right]$$

$$\begin{cases} x + 2z + w + r = 2 & (1) \\ y + w + 2r = 1 & (2) \\ z + \frac{3}{2}w + 2r = \frac{1}{2} & (3) \\ w + 6r = 1 & (4) \end{cases}$$

let $r = r$

$\boxed{w = 1-6r}$

sub w & r in (3)

$z + \frac{3}{2}(1-6r) + 2r = \frac{1}{2} \Rightarrow z + \frac{3}{2} - 9r + 2r = \frac{1}{2}$

$\Rightarrow \boxed{z = -1 + 7r}$

sub w & r in (2)

$y + 1 - 6r + 2r = 1 \Rightarrow \boxed{y = 4r}$

sub z, w & r in (1)

$x + 2(-1+7r) + 1 - 6r + r = 2 \Rightarrow x - 2 + 14r + 1 - 5r = 2$

$\Rightarrow \boxed{x = -9r + 3}$

then, the sol'n of (S): $\{-9r+3, 4r, -1+7r, 1-6r, r\}$

1.2.31

$$\begin{array}{c}
\begin{array}{cccccc} x & y & z & w & r & \end{array} \\
\left[\begin{array}{ccccc|c}
1 & 0 & 2 & 1 & 1 & 2 \\
0 & 1 & 0 & 1 & 2 & 1 \\
0 & 2 & 0 & 0 & 1 & 3 \\
1 & -1 & 2 & 2 & 2 & 0
\end{array}\right]
\end{array}
\quad R_4 \to R_4 - R_1$$

$$\to \left[\begin{array}{ccccc|c}
1 & 0 & 2 & 1 & 1 & 2 \\
0 & 1 & 0 & 1 & 2 & 1 \\
0 & 2 & 0 & 0 & 1 & 3 \\
0 & -1 & 0 & 1 & 1 & -2
\end{array}\right]
\quad \begin{array}{l} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 + R_2 \end{array}$$

$$\to \left[\begin{array}{ccccc|c}
1 & 0 & 2 & 1 & 1 & 2 \\
0 & 1 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & -2 & -3 & 1 \\
0 & 0 & 0 & 2 & 3 & -1
\end{array}\right]
\quad R_4 \to R_4 + R_3$$

$$\to \left[\begin{array}{ccccc|c}
1 & 0 & 2 & 1 & 1 & 2 \\
0 & 1 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & -2 & -3 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\quad R_3 \to -\frac{1}{2} R_3$$

$$\to \left[\begin{array}{ccccc|c}
1 & 0 & 2 & 1 & 1 & 2 \\
0 & 1 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 1 & 3/2 & -1/2 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

(LS) has infinitely many sol'n

x, y & w are the leading variables

z & r are the free variables

then we have to write x, y & w in terms of z & r

$$\begin{cases} x + 2z + w + r = 2 & (1) \\ y + w + 2r = 1 & (2) \\ w + \frac{3}{2}r = -\frac{1}{2} & (3) \end{cases}$$

$$\boxed{w = \frac{-1-3r}{2}}$$

$r = r$ et $w = w$

sub w in (2) $\Rightarrow y - \frac{1}{2} - \frac{3r}{2} + 2r = 1$

$$\boxed{y = \frac{3-r}{2}}$$

sub w in (1)

$x + 2z - \frac{1}{2} - \frac{3r}{2} + r = 2$

$$\boxed{x = -2z + \frac{r}{2} + \frac{5}{2}}$$

then the set of sol'ns of (S) is:

$$\left\{ -2z + \frac{r}{2} + \frac{5}{2},\ \frac{3-r}{2},\ z,\ \frac{-1-3r}{2},\ r \right\}$$

1.2.32 $\begin{cases} 7x + 14y + 15z = 22 \\ 2x + 4y + 3z = 5 \\ 3x + 6y + 10z = 13 \end{cases}$

$\Leftrightarrow \left(\begin{array}{ccc|c} 7 & 14 & 15 & 22 \\ 2 & 4 & 3 & 5 \\ 3 & 6 & 10 & 13 \end{array}\right) R_1 \to \frac{1}{7} R_1$

$\to \left(\begin{array}{ccc|c} 1 & 2 & 15/7 & 22/7 \\ 2 & 4 & 3 & 5 \\ 3 & 6 & 10 & 13 \end{array}\right) \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$

$5 - 2\left(\frac{22}{7}\right)$ $5 - 4\,?$

$3 - 2\left(\frac{15}{7}\right)$

$3 - \frac{30}{7}$

$\frac{14 - 30}{7}$

$10 - 3\left(\frac{15}{7}\right)$

$\to \left(\begin{array}{ccc|c} 1 & 2 & 15/7 & 22/7 \\ 0 & 0 & -9/7 & -9/7 \\ 0 & 0 & 25/7 & 25/7 \end{array}\right) \begin{array}{l} \\ R_2 \to -\frac{7}{9} R_2 \\ R_3 \to \frac{7}{25} R_3 \end{array}$

$\to \left(\begin{array}{ccc|c} 1 & 2 & 15/7 & 22/7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) R_3 \to R_3 - R_2$

$\to \left(\begin{array}{ccc|c} 1 & 2 & 15/7 & 22/7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ (S) has infinitely many sol'ns

x & z are the leading entries

& y is the free entry

$z = 1$, $x + 2y + \frac{15}{7}(1) = \frac{22}{7}$

$x = -2y + 1$

then the set of sol'ns of (S) is $\{-2y+1, y, 1\}$

1.2.33 $\begin{cases} 3x - y + 4z = 6 \\ y + 8z = 0 \\ -2x + y = -4 \end{cases}$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 3 & -1 & 4 & 6 \\ 0 & 1 & 8 & 0 \\ -2 & 1 & 0 & -4 \end{array}\right) \quad R_1 \to \tfrac{1}{3} R_1$$

$$\to \left(\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{4}{3} & 2 \\ 0 & 1 & 8 & 0 \\ -2 & 1 & 0 & -4 \end{array}\right) \quad R_3 \to R_3 + 2R_1$$

$$\to \left(\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{4}{3} & 2 \\ 0 & 1 & 8 & 0 \\ 0 & \frac{1}{3} & \frac{8}{3} & 0 \end{array}\right) \quad R_3 \to R_3 - \tfrac{1}{3} R_2$$

$0+2(\frac{4}{3})$

$\frac{8}{3}$

$-4+2$

$$\to \left(\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{4}{3} & 2 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

(S) has infinitely many sol's

x & y are the leading variables

z is the free variable.

$$\begin{cases} x - \frac{1}{3} y + \frac{4}{3} z = 2 \\ y + 8z = 0 \quad \Rightarrow y = -8z \\ z = z \end{cases}$$

$$x + \frac{8}{3} z + \frac{4}{3} z = 2$$

$$\boxed{x = -4z + 2}$$

Then, the set of sol'ns of (S) is $\{-4z+2, -8z, z\}$

-1
2
-1

1.2.34 $\begin{cases} 9x - 2y + 4z = -17 \\ 13x - 3y + 6z = -25 \\ -2x - z = 3 \end{cases}$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 9 & -2 & 4 & -17 \\ 13 & -3 & 6 & -25 \\ -2 & 0 & -1 & 3 \end{array}\right) \quad R_1 \to \frac{1}{9} R_1$$

$$\to \left(\begin{array}{ccc|c} 1 & -2/9 & 4/9 & -17/9 \\ 13 & -3 & 6 & -25 \\ -2 & 0 & -1 & 3 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - 13R_1 \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\to \left(\begin{array}{ccc|c} 1 & -2/9 & 4/9 & -17/9 \\ 0 & -1/9 & 2/9 & -4/9 \\ 0 & -4/9 & -1/9 & -7/9 \end{array}\right) \quad R_2 \to -9R_2$$

$$\to \left(\begin{array}{ccc|c} 1 & -2/9 & 4/9 & -17/9 \\ 0 & 1 & -2 & 4 \\ 0 & -\frac{4}{9} & -1/9 & -7/9 \end{array}\right) \quad R_3 \to R_3 + \frac{4}{9} R_2$$

$$\to \left(\begin{array}{ccc|c} 1 & -2/9 & 4/9 & -17/9 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & -1 & 1 \end{array}\right)$$

$rk(A) = rk(A|B) = n = 3$
then, the sys has
1 sol'n

so we can obtain the sol'n of this sys by using LU-decomposition

$$U = \begin{pmatrix} 9 & -2 & 4 \\ 13 & -3 & 6 \\ -2 & 0 & -1 \end{pmatrix} \quad \begin{array}{l} R_2 \to R_2 - \frac{13}{9} R_1 \\ R_3 \to R_3 + \frac{2}{9} R_1 \end{array}$$

$$= \begin{pmatrix} 9 & -2 & 4 \\ 0 & -1/9 & 2/9 \\ 0 & -4/9 & -1/9 \end{pmatrix} \quad R_3 \to R_3 - 4R_2$$

$$= \begin{pmatrix} 9 & -2 & 4 \\ 0 & -1/9 & 2/9 \\ 0 & 0 & -1 \end{pmatrix}$$

upper $\triangle$ matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_1 = I_3 \left(R_2 \to R_2 - \frac{13}{9} R_1\right) = \begin{pmatrix} 1 & 0 & 0 \\ -13/9 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow E_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 13/9 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_2 = I_3 \left(R_3 \to R_3 + \frac{2}{9} R_1\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2/9 & 0 & 1 \end{pmatrix} \Rightarrow E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2/9 & 0 & 1 \end{pmatrix}$$

$$E_3 = I_3 \left(R_3 \to R_3 - 4R_2\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{pmatrix} \to E_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

$$L = E_1^{-1} \times E_2^{-1} \times E_3^{-1}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 13/9 & 1 & 0 \\ -2/9 & 4 & 1 \end{pmatrix}$$

$A = LU$ et $AX = b$

$\Rightarrow LUX = b$

$\Rightarrow LY = b$ / $UX = Y$

$$\begin{cases} ① \; LY = b \\ ② \; UX = Y \end{cases}$$

$$A = \begin{pmatrix} 9 & -2 & 4 \\ 13 & -3 & 6 \\ -2 & 0 & -1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad b = \begin{pmatrix} -17 \\ -25 \\ 3 \end{pmatrix}$$

① $LY = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{13}{9} & 1 & 0 \\ -\frac{2}{9} & 4 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -17 \\ -25 \\ 3 \end{bmatrix}$$

$$\begin{cases} y_1 = -17 \\ \frac{13}{9} y_1 + y_2 = -25 \quad \Rightarrow y_2 = -\frac{4}{9} \\ -\frac{2}{9} y_1 + 4 y_2 + y_3 = 3 \quad \Rightarrow y_3 = 1 \end{cases}$$

$$\Rightarrow Y = \begin{bmatrix} -17 \\ -\frac{4}{9} \\ 1 \end{bmatrix}$$

② $UX = Y$

$$\begin{bmatrix} 9 & -2 & 4 \\ 0 & -\frac{1}{9} & \frac{2}{9} \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -17 \\ -\frac{4}{9} \\ 1 \end{bmatrix}$$

$$\begin{cases} 9x - 2y + 4z = -17 \quad ① \\ -\frac{1}{9} y + \frac{2}{9} z = -\frac{4}{9} \quad ② \\ -z = 1 \quad ③ \end{cases}$$

$\Rightarrow \boxed{z = -1}$

sub z in ② $\Rightarrow -y - 2 = -4$ $\boxed{y = 2}$

sub z + y in ① $\Rightarrow \boxed{x = -1}$

$$\Rightarrow X = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

2
4
5

1.2.35 $\begin{cases} 65x + 84y + 16z = 546 \\ 81x + 105y + 20z = 682 \\ 84x + 110y + 21z = 713 \end{cases}$

$$\Rightarrow \left(\begin{array}{ccc|c} 65 & 84 & 16 & 546 \\ 81 & 105 & 20 & 682 \\ 84 & 110 & 21 & 713 \end{array}\right) \begin{array}{l} R_2 \to R_2 - \frac{81}{65} R_1 \\ R_3 \to R_3 - \frac{84}{65} R_1 \end{array}$$

$$\to \left(\begin{array}{ccc|c} 65 & 84 & 16 & 546 \\ 0 & 21/65 & 4/65 & 8/5 \\ 0 & 94/65 & 21/65 & 37/5 \end{array}\right) \quad R_3 \to R_3 - \frac{94}{21} R_2$$

$$\to \left(\begin{array}{ccc|c} 65 & 84 & 16 & 546 \\ 0 & 21/65 & 4/65 & 8/5 \\ 0 & 0 & \frac{1}{21} & \frac{5}{21} \end{array}\right)$$

$\frac{94}{65} \times \frac{21}{65}$

$x = \frac{94}{21}$

$$\begin{cases} 65x + 84y + 16z = 546 & (1) \\ \frac{21}{65}y + \frac{4}{65}z = 8/5 & (2) \\ \frac{z}{21} = \frac{5}{21} & (3) \Rightarrow \boxed{z = 5} \end{cases}$$

sub z in (2)

$\frac{21}{65}y + \frac{4}{65}(5) = \frac{8}{5} \Rightarrow \boxed{y = 4}$

sub z & y in (1) $\Rightarrow \boxed{x = 2}$

then the unique sol'n of S is $x=2, y=4, z=5$

1.2.36 $\begin{cases} 8x + 2y + 3z = -3 \\ 8x + 3y + 3z = -1 \\ 4x + y + 3z = -9 \end{cases}$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 8 & 2 & 3 & -3 \\ 8 & 3 & 3 & -1 \\ 4 & 1 & 3 & -9 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - \frac{1}{2} R_1 \end{array}$$

$$\to \left(\begin{array}{ccc|c} 8 & 2 & 3 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & \frac{3}{2} & -\frac{15}{2} \end{array}\right) \quad (S) \text{ has } 1 \text{ sol'n}$$

$$\begin{cases} 8x + 2y + 3z = -3 \\ y = 2 \\ \frac{3}{2} z = -\frac{15}{2} \Rightarrow \boxed{z = -5} \end{cases}$$

$$8x + 2(2) + 3(-5) = -3$$

$$\Rightarrow \boxed{x = 1}$$

then the sol'n of S is : $x = 1, y = 2, z = -5$

1.2.37 $\begin{cases} -8x + 2y + 5z = 18 \\ -8x + 3y + 5z = 13 \\ -4x + y + 5z = 19 \end{cases}$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -8 & 2 & 5 & 18 \\ -8 & 3 & 5 & 13 \\ -4 & 1 & 5 & 19 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - \frac{1}{2} R_1 \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|c} -8 & 2 & 5 & 18 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & \frac{5}{2} & 10 \end{array}\right)$$ (S) has 1 sol'n

$$\begin{cases} -8x + 2y + 5z = 18 & (1) \\ y = -5 & (2) \\ \frac{5}{2}z = 10 \; (3) \Rightarrow \boxed{z = 4} \end{cases}$$

sub y & z in (1)

$-8x + 2(-5) + 5(4) = 18 \Rightarrow \boxed{x = -1}$

the sol'n of (S) is: $x = -1$, $y = -5$, $z = 4$

1.2.38 $$\begin{cases} 3x - y - 2z = 3 \\ y - 4z = 0 \\ -2x + y = -2 \end{cases}$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 3 & -1 & -2 & 3 \\ 0 & 1 & -4 & 0 \\ -2 & 1 & 0 & -2 \end{array}\right) \quad R_3 \rightarrow R_3 + \frac{2}{3}R_1$$

$$\rightarrow \left(\begin{array}{ccc|c} 3 & -1 & -2 & 3 \\ 0 & 1 & -4 & 0 \\ 0 & \frac{1}{3} & -\frac{4}{3} & 0 \end{array}\right) \quad R_3 \rightarrow R_3 - \frac{1}{3}R_2$$

$$\rightarrow \left(\begin{array}{ccc|c} 3 & -1 & -2 & 3 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$ then this sys has infinitely many sol'ns

$z = z$

$y - 4z = 0 \Rightarrow y = 4z$

$3x - y - 2z = 3 \Rightarrow x = 3z + 1$

the set of sol'ns of (S) is $\{3z+1, 4z, z\}$

1.2.39 $\begin{cases} -9x + 15y = 66 \\ -11x + 18y = 79 \\ -x + y = 4 \\ z = 3 \end{cases}$

$$\Rightarrow \left(\begin{array}{ccc|c} -9 & 15 & 0 & 66 \\ -11 & 18 & 0 & 79 \\ -1 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - \frac{11}{9} R_1 \\ R_3 \to R_3 - \frac{1}{9} R_1 \end{array}$$

$$\to \left(\begin{array}{ccc|c} -9 & 15 & 0 & 66 \\ 0 & -\frac{1}{3} & 0 & -\frac{5}{3} \\ 0 & -\frac{2}{3} & 0 & -\frac{10}{3} \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\begin{cases} -9x + 15y = 66 & (1) \\ -\frac{1}{3}y = -\frac{5}{3} & (2) \\ -\frac{2}{3}y = -\frac{10}{3} & (3) \\ z = 3 \end{cases} \quad (2),(3) \Rightarrow \boxed{y = 5}$$

sub y in (1) $\Rightarrow -9x + 15(5) = 66$

$$\boxed{x = 1}$$

the sol'n of (S) is : $x = 1$, $y = 5$, $z = 3$

rK = ?

1.2.46
$$\begin{bmatrix} 4 & -16 & -1 & -5 \\ 1 & -4 & 0 & -1 \\ 1 & -4 & -1 & -2 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 1/4\, R_1 \\ R_3 \to R_3 - 1/4\, R_1 \end{matrix}$$

$$\to \begin{bmatrix} 4 & -16 & -1 & -5 \\ 0 & 0 & 1/4 & 1/4 \\ 0 & 0 & -3/4 & -3/4 \end{bmatrix} \quad R_3 \to R_3 + 3R_2$$

$$\to \begin{bmatrix} 4 & -16 & -1 & -5 \\ 0 & 0 & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \Rightarrow rK = 2$$

1.2.47
$$\begin{bmatrix} 3 & 6 & 5 & 12 \\ 1 & 2 & 2 & 5 \\ 1 & 2 & 1 & 2 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 1/3\, R_1 \\ R_3 \to R_3 - 1/3\, R_1 \end{matrix}$$

$$\to \begin{bmatrix} 3 & 6 & 5 & 12 \\ 0 & 0 & 1/3 & 1 \\ 0 & 0 & -2/3 & -2 \end{bmatrix} \quad R_3 \to R_3 + 2R_2$$

$$\to \begin{bmatrix} 3 & 6 & 5 & 12 \\ 0 & 0 & 1/3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \Rightarrow rK = 2$$

1.2.48 $\begin{bmatrix} 0 & 0 & -1 & 0 & 3 \\ 1 & 4 & 1 & 0 & -8 \\ 1 & 4 & 0 & 1 & 2 \\ -1 & -4 & 0 & -1 & -2 \end{bmatrix}$ $R_1 \leftrightarrow R_2$

$\to \begin{bmatrix} 1 & 4 & 1 & 0 & -8 \\ 0 & 0 & -1 & 0 & 3 \\ 1 & 4 & 0 & 1 & 2 \\ -1 & -4 & 0 & -1 & -2 \end{bmatrix}$ $R_3 \to R_3 - R_1$, $R_4 \to R_4 + R_1$

$\to \begin{bmatrix} 1 & 4 & 1 & 0 & -8 \\ 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 1 & 10 \\ 0 & 0 & 1 & -1 & -10 \end{bmatrix}$ $R_3 \to R_3 - R_2$, $R_4 \to R_4 + R_2$

$\to \begin{bmatrix} 1 & 4 & 1 & 0 & -8 \\ 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & -1 & -7 \end{bmatrix}$ $R_4 \to R_4 + R_3$

$\to \begin{bmatrix} 1 & 4 & 1 & 0 & -8 \\ 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\Rightarrow rk = 3$

1.2.49 $\begin{bmatrix} 4 & -4 & 3 & -9 \\ 1 & -1 & 1 & -2 \\ 1 & -1 & 0 & -3 \end{bmatrix}$ $R_2 \to R_2 - 1/4\, R_1$, $R_3 \to R_3 - 1/4\, R_1$

$\to \begin{bmatrix} 4 & -4 & 3 & -9 \\ 0 & 0 & 1/4 & 1/4 \\ 0 & 0 & -3/4 & 21/4 \end{bmatrix}$ $R_3 \to R_3 + 3R_2$

$\to \begin{bmatrix} 4 & -4 & 3 & -9 \\ 0 & 0 & 1/4 & 1/4 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ $\Rightarrow rk = 3$

1.2.50 $\begin{bmatrix} 2 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 7 \\ 1 & 0 & 0 & 1 & 7 \end{bmatrix}$ $R_2 \to R_2 - 1/2\, R_1$, $R_3 \to R_3 - 1/2\, R_1$, $R_4 \to R_4 - 1/2\, R_1$

$\to \begin{bmatrix} 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & -1/2 & 1 & 13/2 \\ 0 & 0 & -1/2 & 1 & 13/2 \end{bmatrix}$ $R_3 \to R_3 + R_2$, $R_4 \to R_4 + R_2$

$\to \begin{bmatrix} 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}$ $R_4 \to R_4 - R_3$

$\to \begin{bmatrix} 2 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\Rightarrow rK = 3$

1.2.51 $\begin{bmatrix} 4 & 15 & 29 \\ 1 & 4 & 8 \\ 1 & 3 & 5 \\ 3 & 9 & 15 \end{bmatrix}$

$R_2 \to R_2 - 1/4\, R_1$
$R_3 \to R_3 - 1/4\, R_1$
$R_4 \to R_4 - \frac{3}{4} R_1$

$\to \begin{bmatrix} 4 & 15 & 29 \\ 0 & 1/4 & 3/4 \\ 0 & -3/4 & -9/4 \\ 0 & -9/4 & -27/4 \end{bmatrix}$

$R_3 \to R_3 + 3R_2$
$R_4 \to R_4 + 9R_2$

$\to \begin{bmatrix} 4 & 15 & 29 \\ 0 & 1/4 & 3/4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow rk = 2$